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The Generalized Chain Code for Map Data Encoding and Processing,

by

Herbert/Freeman

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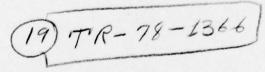




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#### ABSTRACT

The concept of chain coding for map data based on the well-known 8-direction coding matrix is generalized to coding schemes involving 16, 24, 32, 48 and even more permissible directions for the line segment links in the chain representation. General methods for quantization and encoding are described. The different schemes are compared with respect to compactness, precision, smoothness, simplicity of encoding, and facility for processing. The resulting coding schemes appear to have desirable characteristics for map data processing applications because of their improved storage efficiency, smoothness, and reduced processing time requirements.

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### ACKNOWLEDGMENT

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#### 1. INTRODUCTION

In the chain coding scheme for the computer representation of linedrawing data, an overlaid square lattice is assumed and the lines of the drawing are represented by sequences of straight-line segments connecting nodes of the lattice lying closest to the lines. In passing from one node to the next, there are 8 allowed directions, and the concatenated line segments are all of length 1 or  $\sqrt{2}$  (times the lattice spacing). The scheme has been found especially useful for representing free-form line drawing-data as is encountered in geographic maps. It has found wide acceptance for the purpose of digital data transmission and computer processing, mainly because of its inherent simplicity and the ease with which efficient prood for it [1,2]. We shall here show cessing algorithms can be de 'n code can be generalized to that the basic (i.e., 8-direction codes having a much larger number of llowed directions and that such codes, in spite of their increased complexity, may have definite advantages for certain applications [3].

In selecting a line-drawing coding scheme for a particular application, it is helpful to evaluate the scheme against the following five criteria: (1) compactness, (2) precision, (3) smoothness, (4) ease of encoding and decoding, and (5) facility for processing. The relative weight to be assigned to each of these criteria is very much dependent on the intended application. If the purpose of the encoding is primarily storage or transmission, compactness is likely

to be of paramount importance as it directly determines the required amount of computer memory or channel bandwidth (or transmission time). Precision is important if quantitative aspects of the encoded data are of particular interest (i.e., a geographic map). Smoothness may be of significance if the encoded data is ever to be displayed, especially if the "fairness" of a curve is important or if the result is to be aesthetically pleasing.

The weight to be given to ease of encoding (and decoding) will be high if large data quantities are to be encoded. For applications involving smaller data quantities but extensive processing, simplicity of the processing task is likely to outweigh simplicity of encoding.

#### 2. GENERALIZED CHAIN CODES

In the basic (8-point) chain code, the next node (r, s) in sequence for a given present node (i, j) must be one of the 8 nodes that are 1- or  $\sqrt{2}$ -distant, i.e., such that max. |r-i|, |s-j| = 1. Thus in Fig. 1, for a given node A, the permissible next nodes in the basic chain code are the nodes numbered 0 through 7. All of these nodes lie on the boundary of a square of side 2 and centered at A. We shall refer to this square boundary as "ring 1".

Let us now consider a coding scheme in which the "next" node may be any node in ring 1 or in ring 2. (Ring 2 consists of nodes 8 through 23 in Fig. 1). These are the nodes for which max.  $|\mathbf{r}-\mathbf{i}|$ ,  $|\mathbf{s}-\mathbf{j}|=1$  or 2. A chain based on such a 24-point scheme may contain links of length 1,  $\sqrt{2}$ , 2,  $\sqrt{5}$ , and  $2\sqrt{2}$ . Also there will be a total of 16 allowed directions (determined by the nodes of ring 2). A curve

encoded with this scheme is likely to exhibit finer angular quantization and to contain fewer segments than one encoded in the 8-point scheme. Finer angular quantization will permit improved smoothness. Fig. 2 shows a curve encoded in both the 8-point scheme (2a) and the 24-point scheme (2b); the map of Fig. 2c is shown 8-point encoded in Fig. 2d.

A variety of other chain coding schemes can now be readily postulated. However, let us first look at some of the properties of the rings. Examination of Fig. 1 shows that as we advance from ring n to ring n+1, for each non-corner node of ring n, there is a corresponding non-corner node in ring n+1. For each of the 4 corner nodes of ring n, there is a corresponding corner node in ring n+1 as well as 2 non-corner nodes not present in ring n. Hence the total number of nodes in ring n+1 will be greater by 8 than the number of nodes in ring n. Since the number of nodes in ring 1 is 8, it follows that ring n will contain precisely 8n nodes. The number of nodes for all rings, 1 through n inclusive, is 4n(n+1).

In the first octant (slope 0 through 1) the permissible slopes for the set of rings 1 through n are all those that correspond to the rational numbers between 0 and 1 inclusive whose denominators are less than or equal to n, and these, if ordered, are the terms of the Farey series of order n [4,5]. The slopes for the other octants follow from symmetry. The total number of different permissible directions for the set of rings 1 through n is given by 8F(n) - 8, where F(n) is the number of terms in the Farey series of order n.

In forming a chain coding scheme, we may use any number of rings, in any combination. Thus we may from a chain code based solely on ring 2. It will have 16 permissible directions and its links will be of length 2,  $\sqrt{5}$ , and  $2\sqrt{2}$ . Its angular quantization will be either  $18.4^{\circ}$ or 26.5°. It differs from the 24-point code in that steps of length 1 or  $\sqrt{2}$  are not allowed. As a result there may be difficulty in obtaining a closed chain to correspond to a closed curve; that is, the end points of a 16-point encoded chain may be 1 or √2 units apart without the availability of links of such lengths for closing this gap. For example, if one draws in Fig. 1 a line segment from node A to node 23 and from node 23 to node 1 (both permissible 16-point line segments), the end points, nodes A and 1, will be a distance  $\sqrt{2}$  apart. Although this lack of completeness may be objectionable to the purist, in practice it is of minor consequence since a chain can always be closed by some sacrifice in precision. Thus for the previous 2-link chain, drawing the second link from node 23 to node 9 instead of to node 1 will permit closing the chain with a link from 9 to A. The 16-link scheme has been previously proposed for use with digital plotters [6].

In returning to the 24-link code we note that (since the 8-link code is subsumed it) it has all the features of the 8-link code of being able to follow fine detail (small radii of curvature) with short segments but in addition has longer segments for "taking bigger steps" where the curvature is not as severe. These larger steps can

be taken with an angular quantization roughly twice as fine as that of the 8-link scheme. Clearly, with the foregoing in mind, a 48-link scheme utilizing rings 1, 2 and 3 should be even better.

The coding matrices corresponding to 4-,8-,16-,24-,32- and 48-link codes are shown in Fig. 3. Note that the coding matrix for the 32-link consists of rings 1 and 3. This code thus has the ability to take relatively long, fine-angle steps but, because of ring 1, can also follow small detail in a curve. The 48-link code of Fig. 3 (g) consists of the complete rings 1 and 2, and the partial ring 4. In ring 4, those nodes for which one coordinate has value 3 have been omitted. If ring 2 were also eliminated, a 32-link code would result (consisting now of ring 1 and the partial ring 4) that would have an excellent long-distance capability and yet retain the ability to follow fine detail. The rules governing the node relations for the codes in Fig. 3 are shown in Fig. 4.

#### 3. QUANTIZATION

One of the appealing features of the 8-link code has been its simplicity - for quantization, for encoding, and for processing. As we go to higher-order link codes, the complexity of these tasks increases. Let us examine first the quantization problem. In Fig. 5 (a), the so-called grid quantization method for the 8-link code is illustrated. One traces along the curve, and at each intersection between curve and superimposed grid, the node closest to the intersection is selected as next node. The method assures that on average approximately

41 per cent of the links in a chain will be of length √2. An alternate quantization scheme is the so called square-box scheme of Fig. 5 (b), where the next node is selected on the basis of a square box "capture area" surrounding each node. The latter scheme, however, yields then only 4-point coded chains [2].

In Fig. 5 (c) we show how the grid-intersection scheme has been extended to the 24-point code. In determining the next node, one first looks for the intersection between the curve and ring 2. The closest ring-2 node is identified; however, before it can be taken as the next node, it is necessary to determine whether the curve intersects ring 1 within limits set by the grid midpoints to either side of the identified ring-2 node. In Fig. 5 (c), for curve A the ring-2 node is 17. Its limits in ring 1 are located at the 1/4 and 3/4 points between nodes 1 and 2 (note the dashed lines). If the curve intersects ring 1 within these limits, the ring 2 node is the valid next node. Thus in Fig. 5 (c), node 17 is a valid next node for curve A, but node 9 is not a valid next node for curve B. For curve B, the next node must be taken from ring 1.(It will be node 1).

The quantization scheme for the 32-point code (based on rings 1 and 3) is shown in Fig. 5(d). Appropriate limits must be satisfied for rings 3, 2, 1 (in that order). In the figure, curve A satisfies all limits associated with node 9 and node 9 thus becomes the next node. However, node 16 cannot be selected for curve B because the associated ring-2 and ring-1 limits are not satisfied. One should note that, although the 32-point code utilizes only rings 1 and 3, for the purpose

of quantization, <u>all</u> rings of lower order must be considered. The quantization procedure for higher-order codes is similar.

### 4. ENCODING

For the 8-point chain code, the coding convention is well known and is shown in Fig. 6 (b). In Fig. 6 (a) we show the corresponding convention for the 4-point chain code. Possible conventions for the 16- and 24-point codes are shown in Fig. 6 (c) and (d), respectively. For both of the latter codes, addition of 2 to each code value will cause a 90-degree counter-clockwise rotation (subject to appropriate limit checks to assure remaining in the same ring). Some different coding conventions are illustrated in Fig. 7 (a) and (b), which have some advantages over those of Fig. 6. Proposed coding assignments for the 32-point and the two 48-point codes of Fig. 3 are shown in Figs. 8 and 9.

## 5. COMPARATIVE CODE CHARACTERISTICS

There are five major criteria for evaluating the effectiveness of a coding scheme for line-drawing data: precision, compactness, smoothness, simplicity of encoding, and facility for processing. The relative weights to be given to each of these criteria depends somewhat on the application. Where large quantities of line drawing data are involved, as with geographic maps, compactness of storage is an important consideration. Smoothness is important only where visual display is involved. Facility for processing takes on significance if the encoded line drawing data is to be subject to extensive analysis and manipulation.

A sample contour is illustrated in Fig. 10. A square lattice has been overlaid. The lattice spacing is approximately 1/40 of the maxi-

mum distance between two points on the contour. Chain codes were generated using the 4-, 8-, 16-; 24-, and 32-point schemes. The results are given in Fig's. 11 and 12. The disadvantages of the 4-point scheme are at once apparent: it leads to a very coarse contour representation and - for that reason - to a perimeter that is excessively long. The 8-point code gives a moderately good result; however, the higher-order codes are able to yield much smoother perimeters because of their finer angular resolution.

A quantitative comparison of the different codes is given in Table I. The total number of bits for encoding the contour in each code is determined on the basis that a full code word be assigned to each link type of the code (i.e., no code compression using differencing or other techniques is considered). The 16- and 32-point codes are shown to yield considerably more compact representations. (Subsequent code compression would tend to favor the 32-point code over the 16-point code). Precision, as measured here in terms of perimeter length and enclosed area gives the best rating to the 32-point code, though the performance of the other codes (except the 4-point code) is not far behind. Very important here, however, is the total number of links since this directly determines the processing time for virtually any analysis or manipulation algorithm. This strongly favors the 32-point code (38 to 87 as against the 8-point code).

The example shows the potential advantages of the higher-order chain codes in achieving smoother and more compact representations as well as reduced processing time. The advantages appear strong enough to outweigh the increased encoding complexity cost in most applications.

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Code System	Number of Links	Bits per Link	Number of Bits	Length	Area
4	124	2	248	124.0	312.0
8	87	3	261	103.2	310.5
16	44	4	176	97.4	306.5
24	48	5	240	98.1	301.8
32	38	5	190	97.4	307.0
Original	curve			96.5	310.3

Table 1. Results of encoding contour of Fig. 10 in different chain codes.

33	32	31	30	29	28	27
34	14	13	12	11	10	26
35	15	3	2	1	9	25
36	16	4	A (i,j)	0	8	24
37	17	5	6	7_	23	47
38	18	19	20	21	22	46
39	40	41	42	43	44	45
	34 35 36 37	34 14 35 15 36 16 37 17	34 14 13 35 15 3 36 16 4 37 17 5 38 18 19	34 14 13 12  35 15 3 2  36 16 4 A (i,j)  37 17 5 6  38 18 19 20	34 14 13 12 11  35 15 3 2 1  36 16 4 A 0  (i,j)  37 17 5 6 7  38 18 19 20 21	34 14 13 12 11 10  35 15 3 2 1 9  36 16 4 A 0 8  (i,j)  37 17 5 6 7 23  38 18 19 20 21 22

Fig. 1. The different node rings surrounding the given node A: 0 - 7 (ring 1), 8 - 23 (ring 2), 24 - 47 (ring 3), etc. Ring 1 is shown bold.

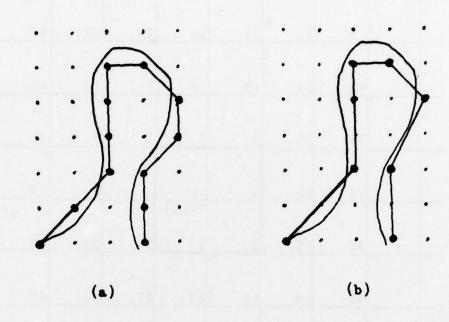


Fig. 2. Two different chain encodings of the same curve: (a) 8-point code, (b) 24-point code.



PHOTOGRAPH OF CONTOUR MAP (FALL RIVER PASS, COLORADO)

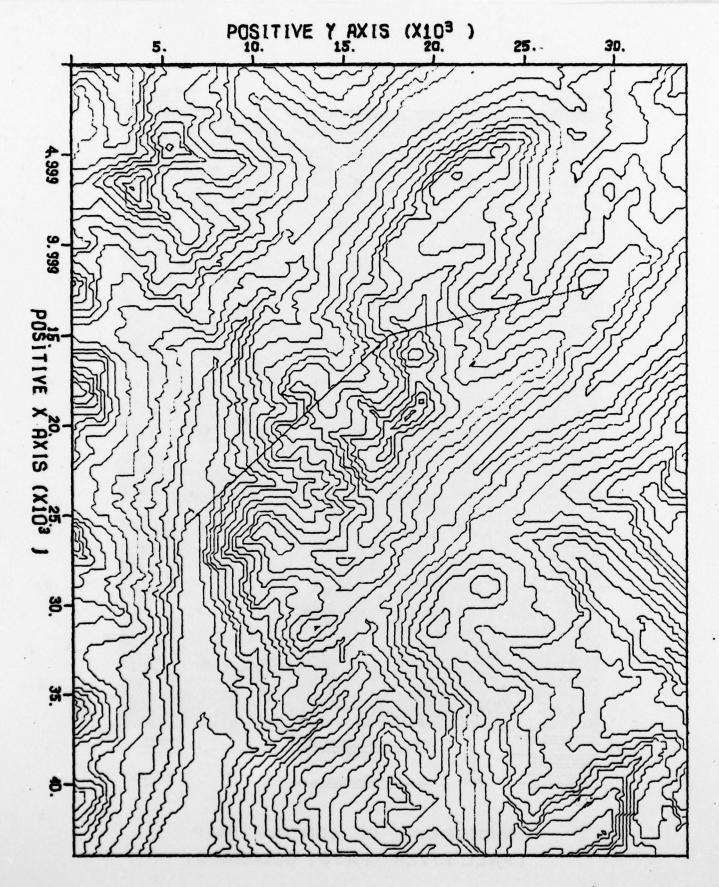


Figure 2d

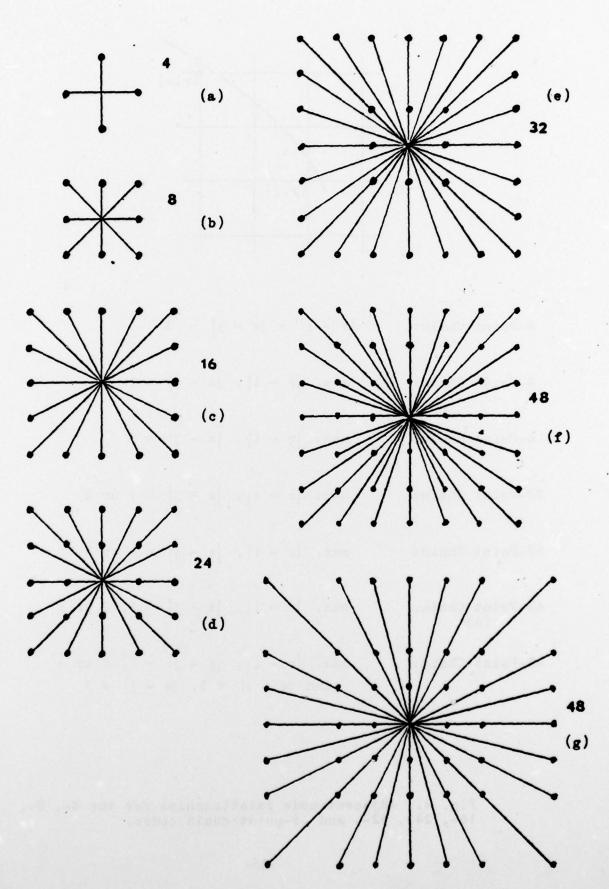
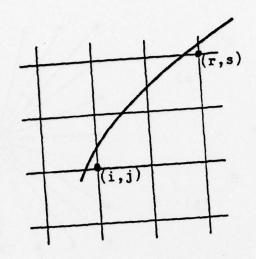


Fig. 3. Coding matrices for the 4-, 8-, 16-, 24-, 32-, and 48-point chain codes. (Two different versions of the 48-point code are shown).



$$|\mathbf{r} - \mathbf{i}| + |\mathbf{s} - \mathbf{j}| = 1$$

max. 
$$|r - i|$$
,  $|s - j| = 1$ 

max. 
$$|r - i|$$
,  $|s - j| = 2$ 

max. 
$$|r - i|$$
,  $|s - j| = 1$  or 2

max. 
$$|r - i|$$
,  $|s - j| = 1$  or 3

max. 
$$|r - i|$$
,  $|s - j| = 1$ , 2 or 3

max. 
$$|r - i|$$
,  $|s - j| = 1$ , 2 or 4  
and  $|r - i| \neq 3$ ,  $|s - j| \neq 3$ 

Fig. 4. Adjacent-node relationships for the 4-, 8-, 16-, 24-, 32-, and 48-point chain codes.

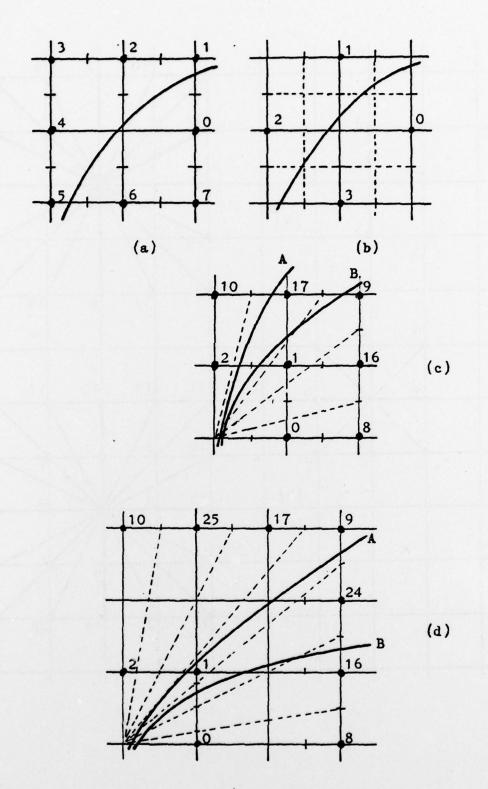


Fig. 5. Quantization schemes for the 4-, 8-, 24-, and 32-point chain codes. (a) 8-point grid-intersect quantization, (b) 4-point square-box quantization, (c) 24-point grid-intersect quantization, and (d) 32-point grid-intersect quantization.

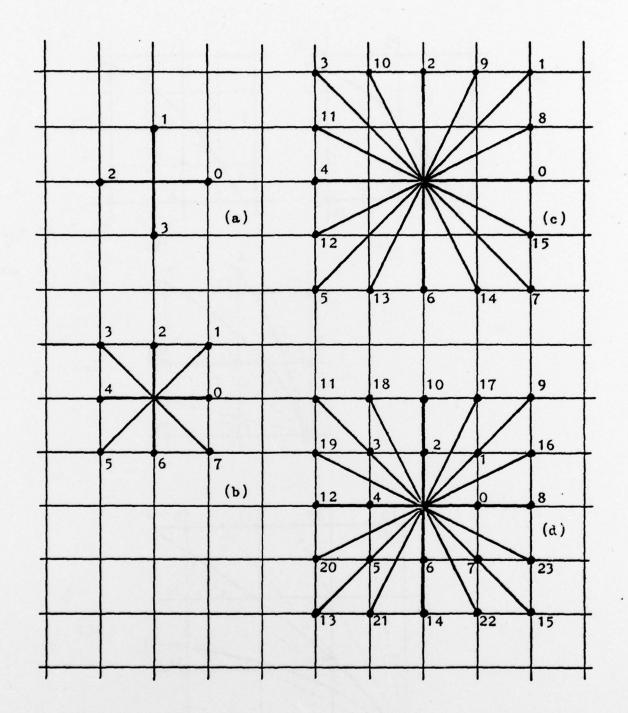


Fig. 6. Code assignments for the 4-, 8-, 16-, and 24-point chain codes.

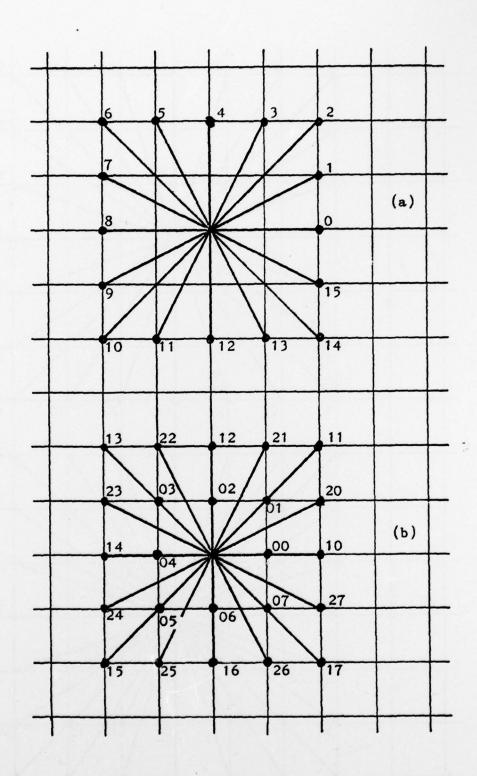


Fig. 7. Alternate code assignments for the 16- and 24-point codes.

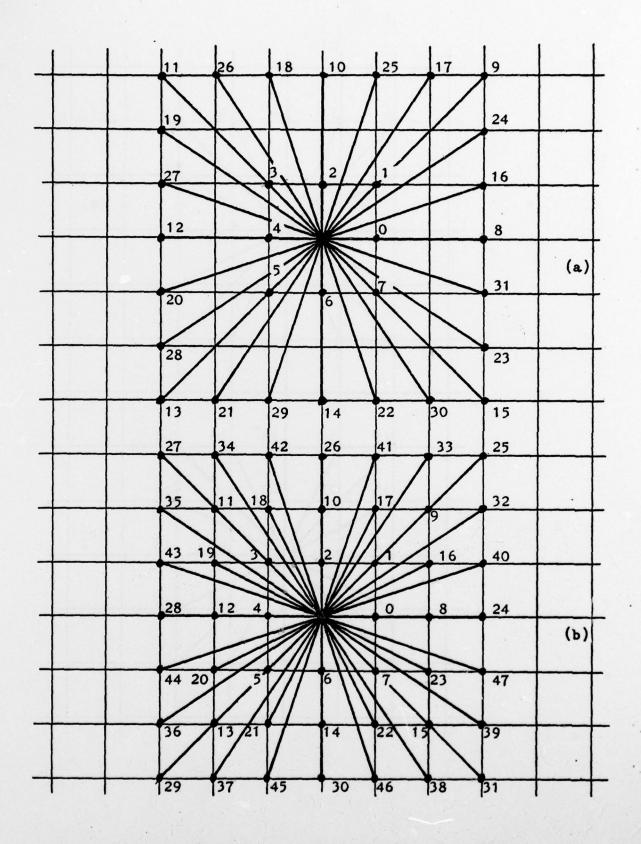


Fig. 8. Code assignments for the 32- and 48-point chain codes.

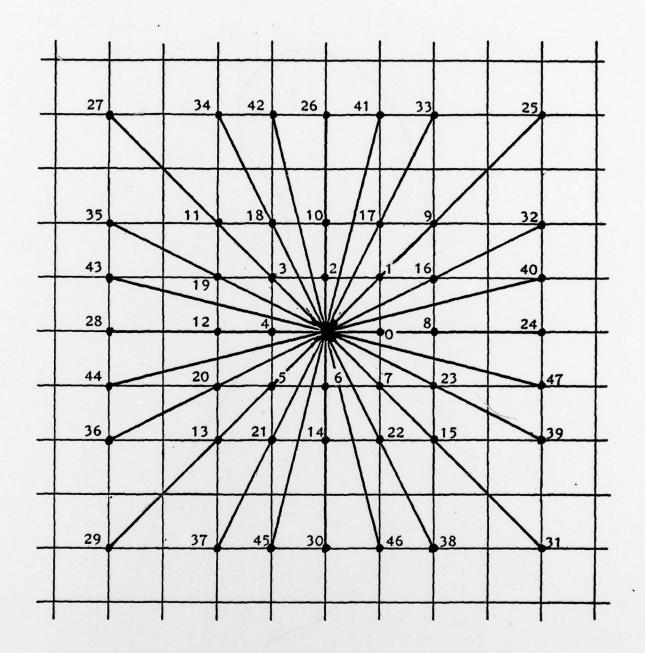


Fig. 9. 48-point chain code constituted from rings 1, 2, and a partial ring 4.

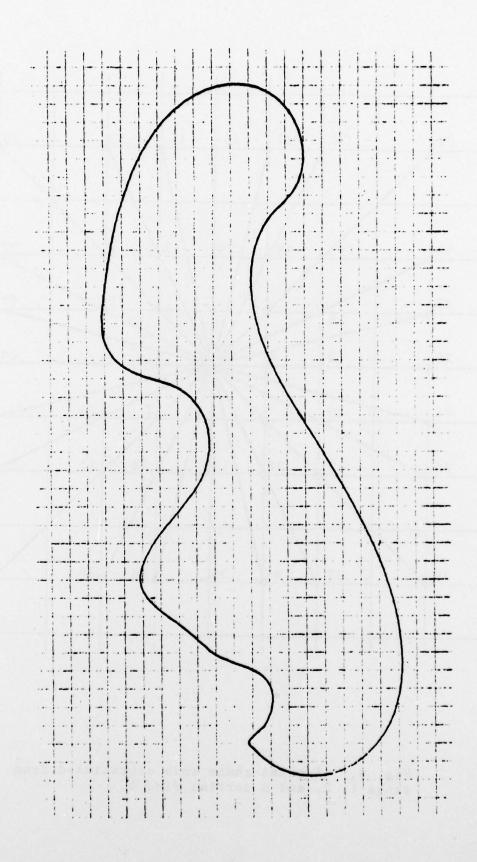


Fig. 10. Sample contour with overlaid square lattice.

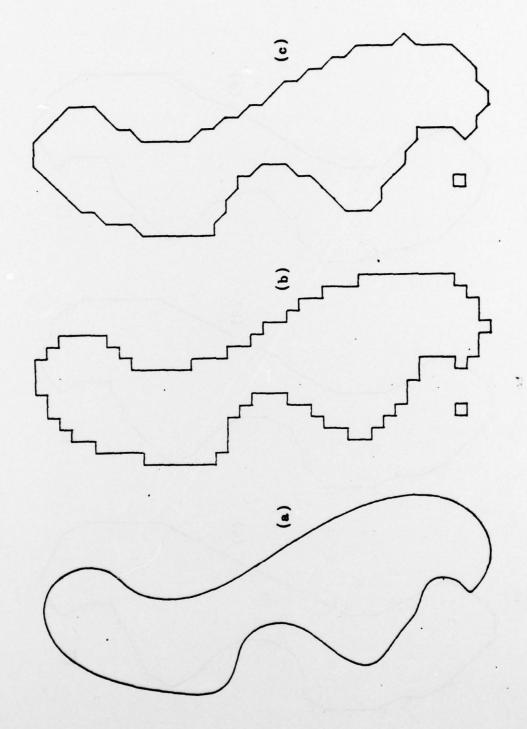


Fig. 11. Comparison of (a) original contour, (b) 4-point chain, and (c) 8-point chain.

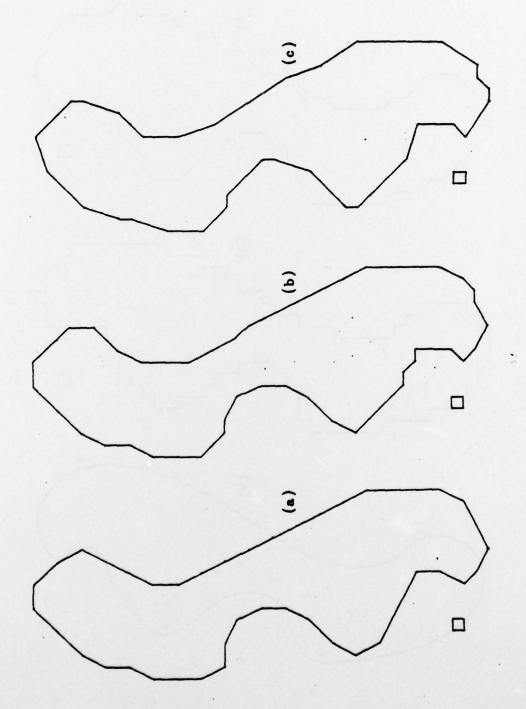


Fig. 12. Comparison of (a) 16-point chain, (b) 24-point chain, and (c) 32-point chain of contour shown in Fig. 10.

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The concept of chain coding for map data based on the well-known 8-direction coding matrix is generalized to coding schemes involving 16, 24, 32, 48 and even more permissible directions for the line segment links in the chain representation. General methods for quantization and encoding are described. The different schemes are compared with respect to compactness, precision, smoothness, simplicity of encoding, and facility for processing. The resulting coding schemes appear to have desirable characteristics for map data processing applications because of their improved storage (over)

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